Unfolding of flow

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Outline

- Setup: Background & Method
- 1D unfolding result
- 2D unfolding result
- ~~3D unfolding result~~ (Running now.)

Setup

Background

- Traditionally, people believe that flow vector should be boost invariant:

$$\vec{v}_n(\eta_a) = \vec{v}_n(\eta_b) = \vec{v}_n(\eta_0)$$

$$\vec{v}_n(\eta_a) \rightarrow \vec{v}_{n,a};\ \vec{v}_n(\eta_b) \rightarrow \vec{v}_{n,b}$$

- However, recent experiments suggest a decorrelation effect between different η segment:

$$\vec{v}_{n,a} = \vec{v}_{n,0} + \vec{d}_{n,a}$$

$$\vec{v}_{n,b} = \vec{v}_{n,0} + \vec{d}_{n,b}$$

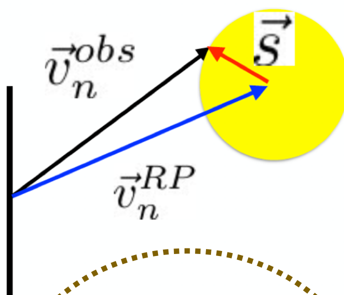
Total flow vector

Decorr. term

$\vec{v}_n(\eta_0)$

Background

- The observed data has a smearing effect(both statistical and non-flow):



Observed flow vector

$$\vec{q}_{n,a} = \vec{v}_{n,0} + \vec{d}_{n,a} + \vec{s}_{n,a}$$

$$\vec{q}_{n,b} = \vec{v}_{n,0} + \vec{d}_{n,b} + \vec{s}_{n,b}$$

Smear term

Step-by-step problem

3D unfolding with decorrelation (our aim)

Toy decorrelation model:
1. No decor.
2. Decor. on EP angle
3. **Decor. on EP angle and magnitude**

Unfolding dimension:
1. **1D**
2. **2D**
3. 3D

(More dimensions, more information.)

1D with no decor. 👉 M. Nie and Arabinda et al 's previous work

Toy decorrelation model

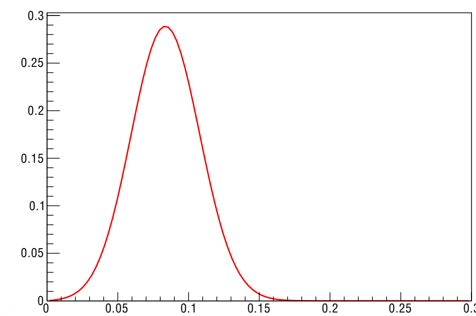
Observed flow vector

$$\vec{q}_{n,a} = \vec{v}_{n,0} + \vec{d}_{n,a} + \vec{s}_{n,a}$$
$$\vec{q}_{n,b} = \vec{v}_{n,0} + \vec{d}_{n,b} + \vec{s}_{n,b}$$

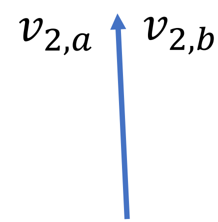
Smear term

$$v_2 \sim \frac{1}{2\pi\delta_2^2} v_2 e^{-\frac{(v_2)^2+\bar{v}_2^2}{2\delta_2^2}} I_0\left(\frac{v_2\bar{v}_2}{\delta_2^2}\right); \sqrt{2}\delta_2 = 0.035, \bar{v}_2 = 0.08$$

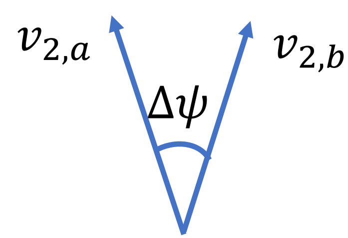
PDF of BG distribution



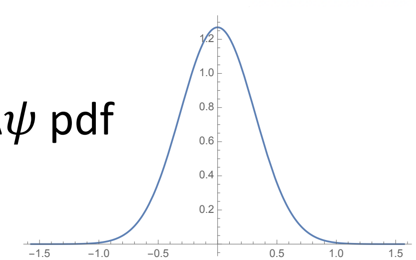
No decor. $v_{2,a} = v_{2,b}, \Delta\psi = 0$



on EP angle $v_{2,a} = v_{2,b}, \Delta\psi \sim N(0, \frac{\pi}{10})$



$\Delta\psi$ pdf



P. Bożek et al. PRC 83, 034911(2001)

Toy decorrelation model (Cont.)

Decor. on EP angle and magnitude

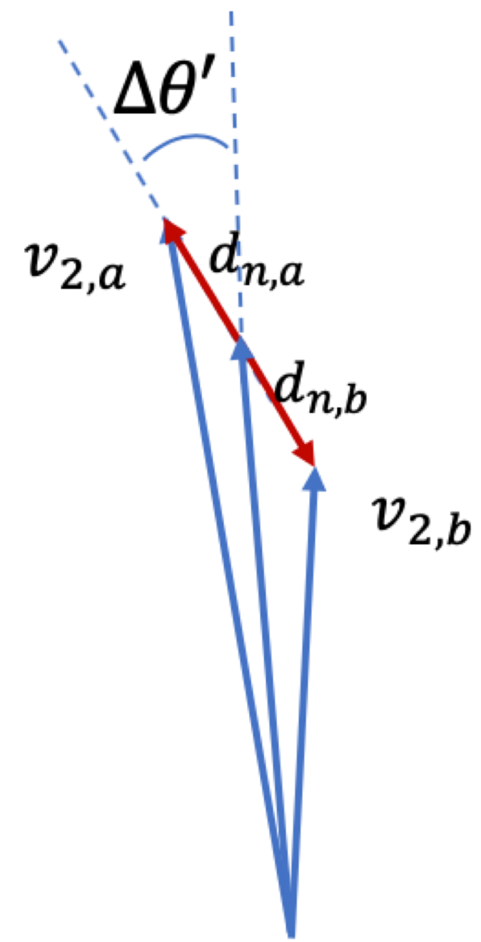
$d_{n,a} = d_{n,b} = 0.02$ (Magnitude)

$\Delta\theta' \sim \text{Uniform}(0,2\pi)$

By this construction, we ensure:

$$\frac{\langle v_{2,a} v_{2,b}^{\star} \rangle}{\langle v_{2,0}^{2} \rangle} \approx 1 - 5.5\%$$

$$\langle v_{2,a} \rangle = \langle v_{2,b} \rangle = \langle v_{2,0}^{2} \rangle$$



Dimension choice

$$\begin{pmatrix} \vec{v}_{2,a} \\ \vec{v}_{2,b} \end{pmatrix} \Longrightarrow \begin{pmatrix} v_{2,a}+v_{2,b} \big/ 2 \\ |v_{2,a}-v_{2,b}| \big/ 2 \\ |\Delta\psi| \end{pmatrix}$$

Dimension	Range	# bins
$v_{2,a}+v_{2,b} \big/ 2$	$0 \sim 0.3$	50
$\lvert v_{2,a}-v_{2,b} \rvert \big/ 2$	$0 \sim 0.07$	10
$\lvert \Delta\psi \rvert$	$0 \sim \pi$	10

Method

- Unfold method: Bayesian Unfolding.
- Implement: RooUnfold Package.

Calculate by iteration

Known

Iteration process

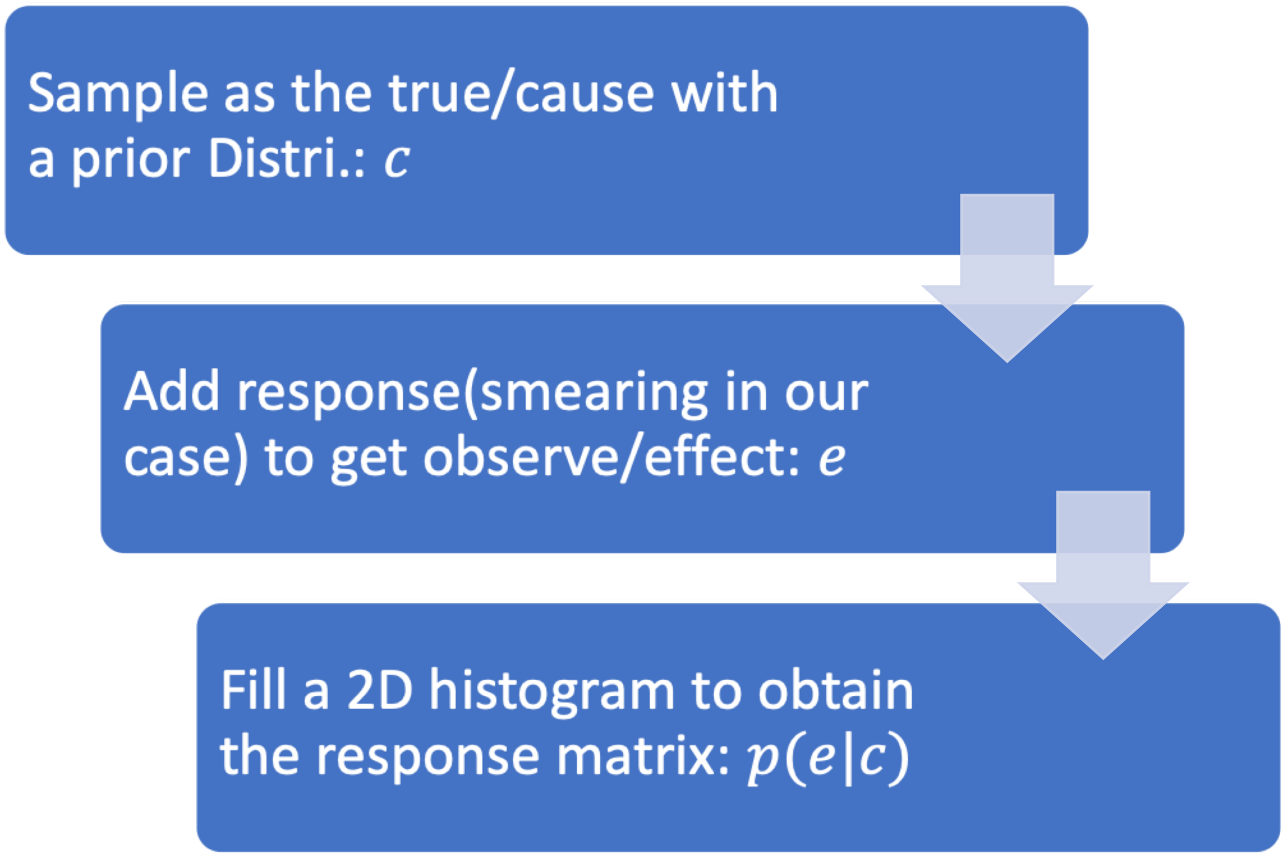
$$\hat{c} : cause(true); \hat{e} : effect(observation), A_{ij} = Count(e_i | c_j)$$

$$\hat{c}^{\text{iter}+1} = \hat{M}^{\text{iter}} \hat{e}, \quad M_{ij}^{\text{iter}} = \frac{A_{ji} c_i^{\text{iter}}}{\sum_{m,k} A_{mi} A_{jk} c_k^{\text{iter}}} .$$

- This method has been employed in several 1D unfolding projects successfully.

Method

The response matrix generation workflow



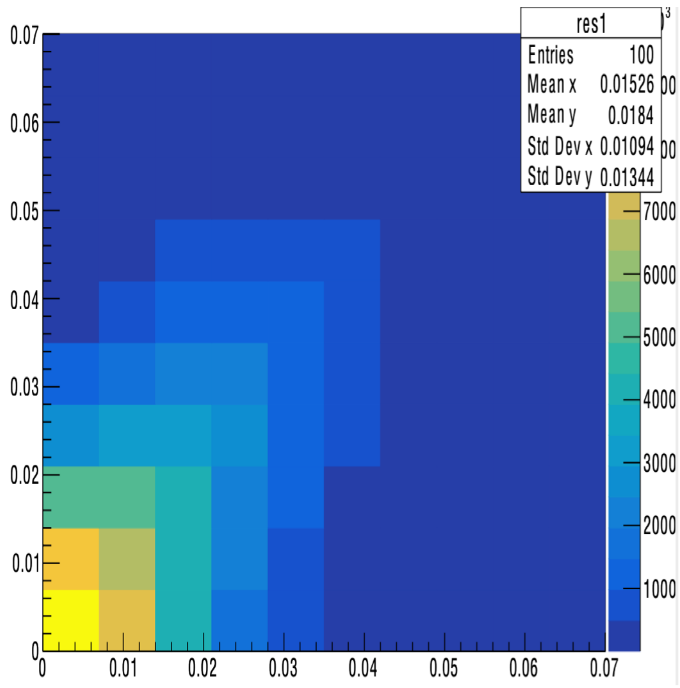
Choice of prior distribution

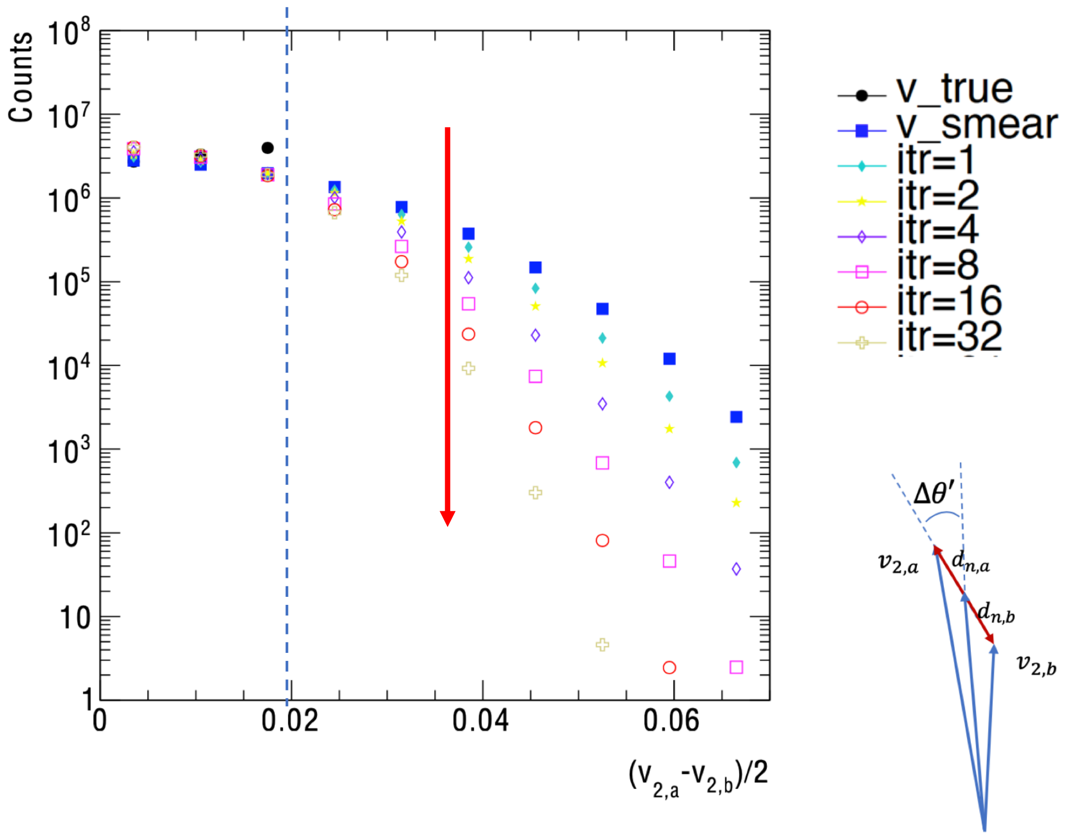
- The unfolded distribution will by construction **be include in** the given prior distribution of response matrix.
- So the prior distribution should at least fully cover unfolded distribution.
- It's wise to choose distribution after smearing as prior.
- In previous results, we chose a prior distribution on a much larger region. And consequently, fewer data on the concerned region, more error on some region.

1D unfolding

1D: $(v_{2,a} - v_{2,b})/2$

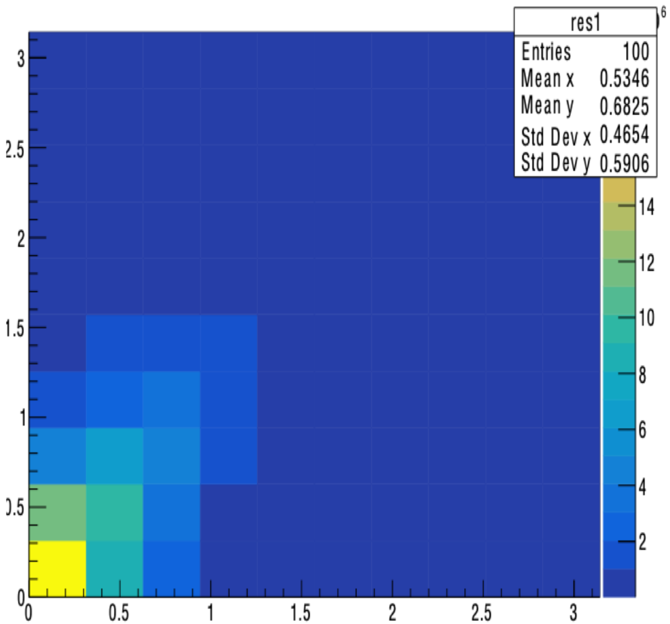
Response matrix

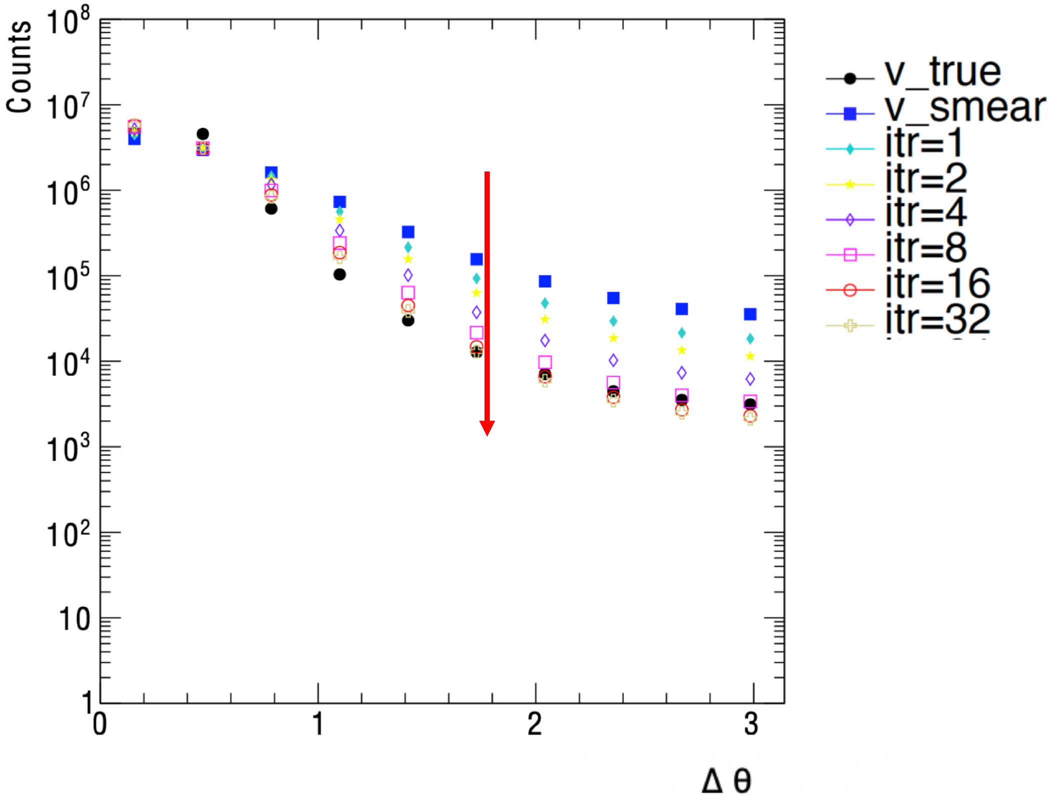




1D: $\Delta\psi$

Response matrix

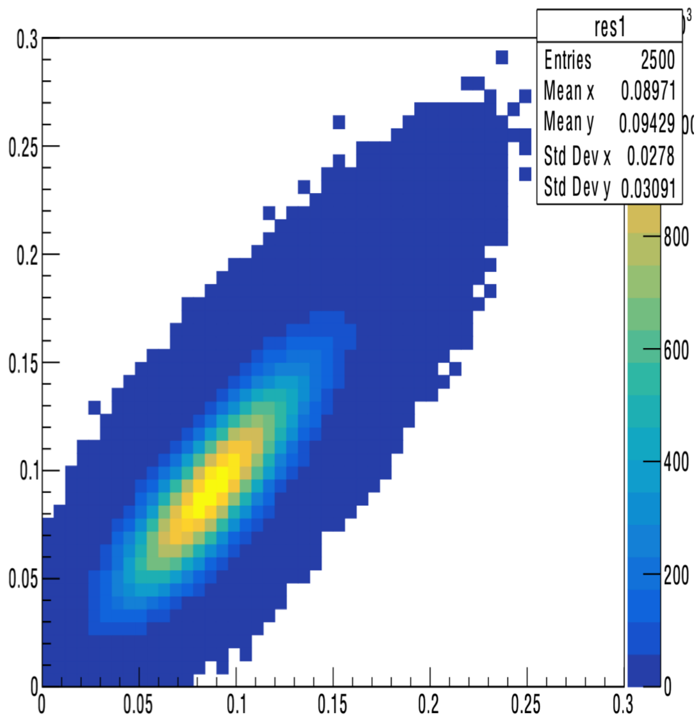


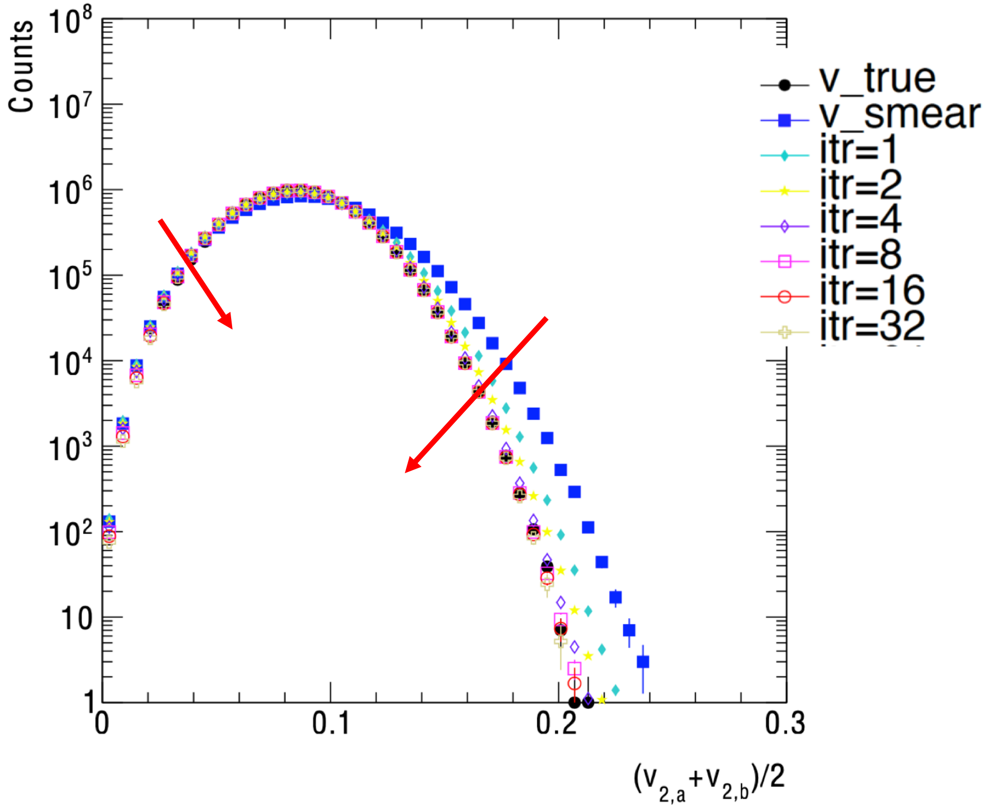


$$\Delta\psi \sim \frac{\text{decorrelation}}{v_2}$$

1D: $(v_{2,a} + v_{2,b})/2$

Response matrix

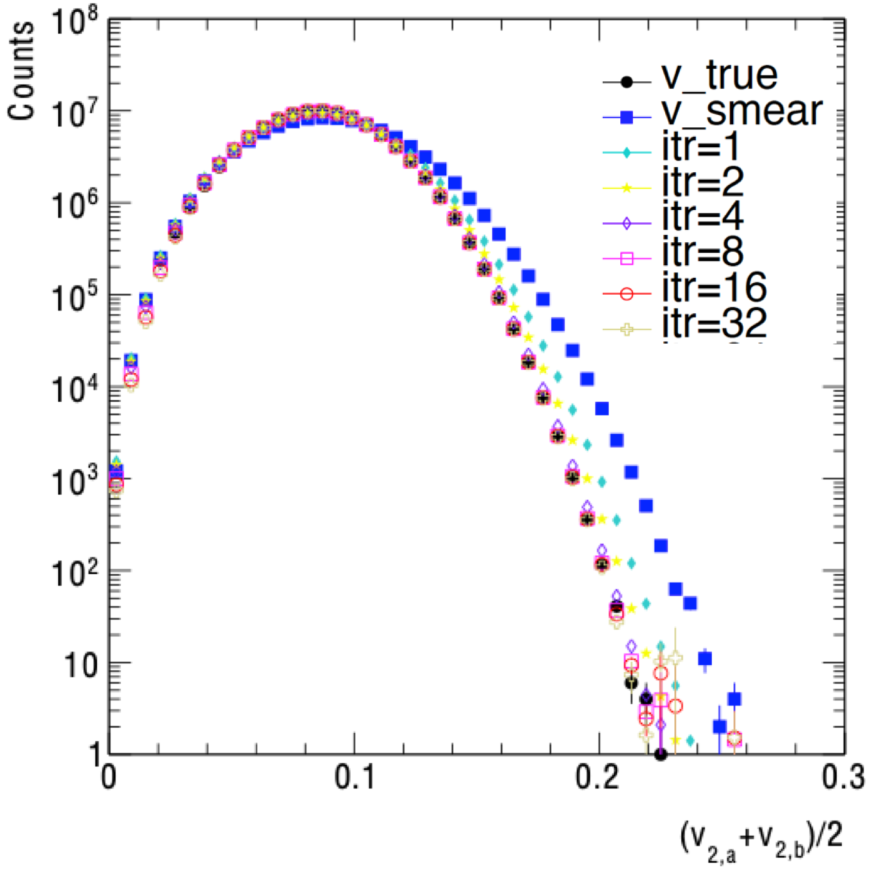




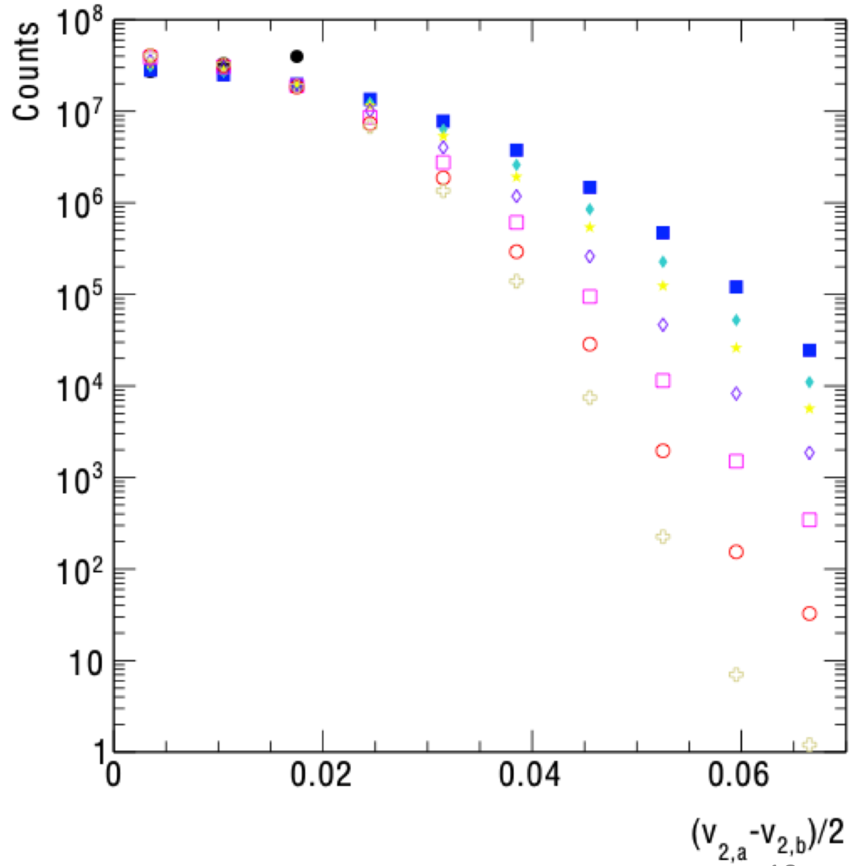
2D unfolding

Unfold joint $\frac{v_{2,a}+v_{2,b}}{2}$ & $\frac{|v_{2,a}-v_{2,b}|}{2}$

Projected on $\frac{v_{2,a}+v_{2,b}}{2}$

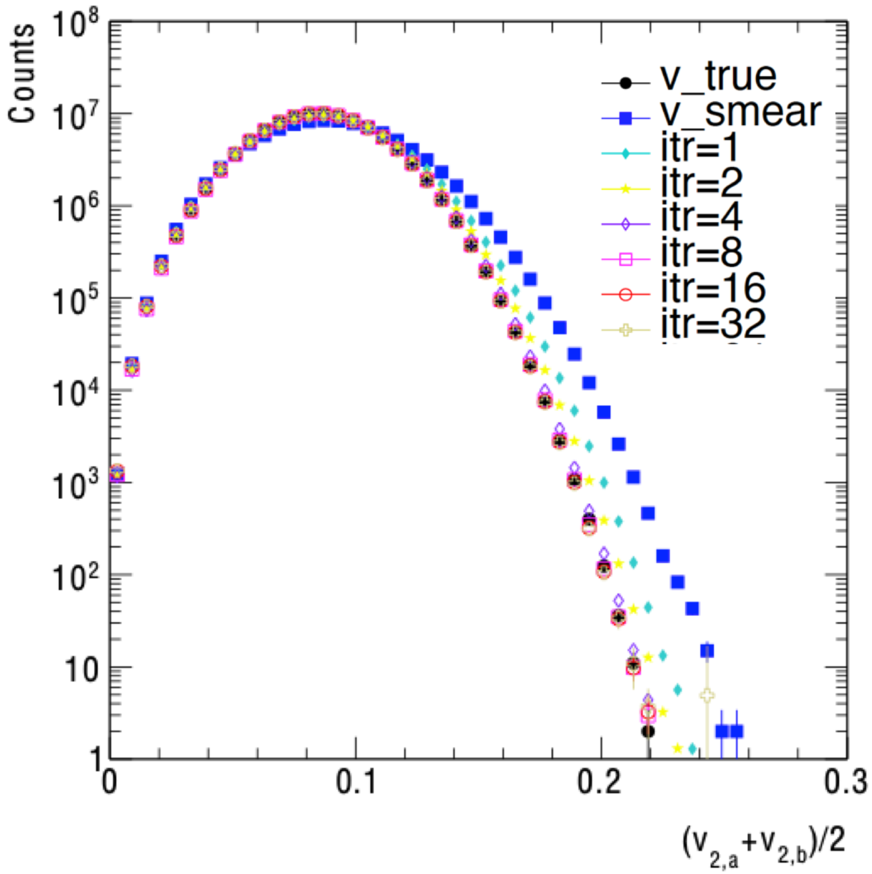


Projected on $\frac{|v_{2,a}-v_{2,b}|}{2}$

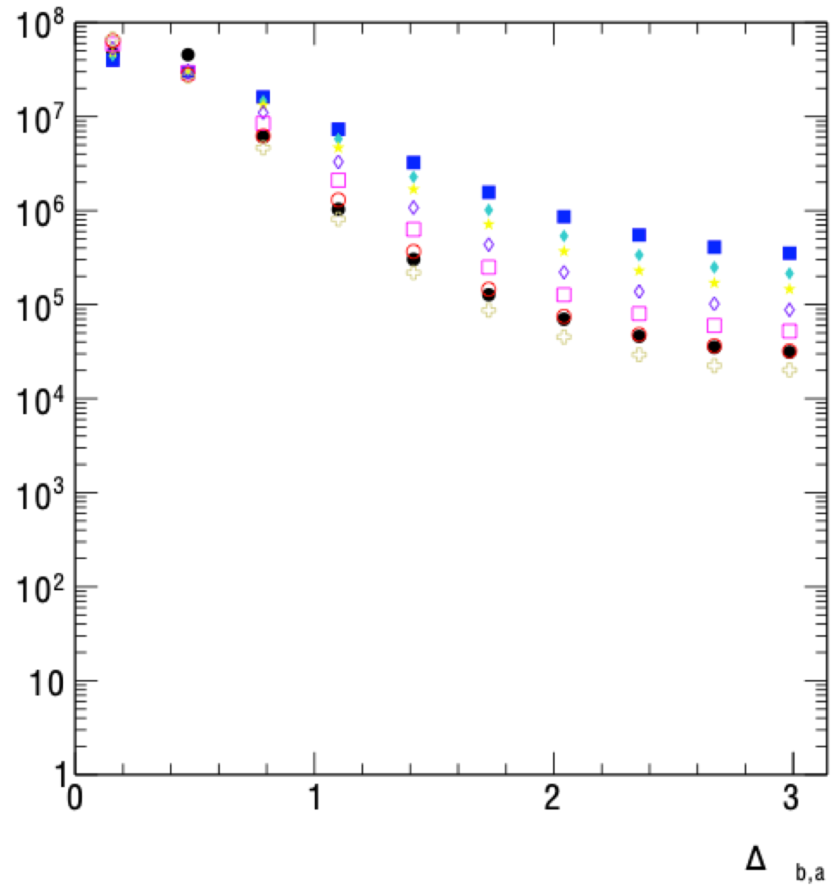


Unfold joint ${v_{2,a}+v_{2,b}}/{2}$ & $\Delta\psi$

Projected on ${v_{2,a}+v_{2,b}}/{2}$



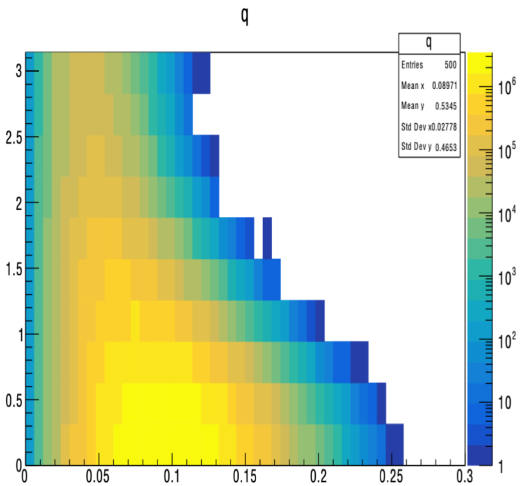
Projected on $\Delta\psi$



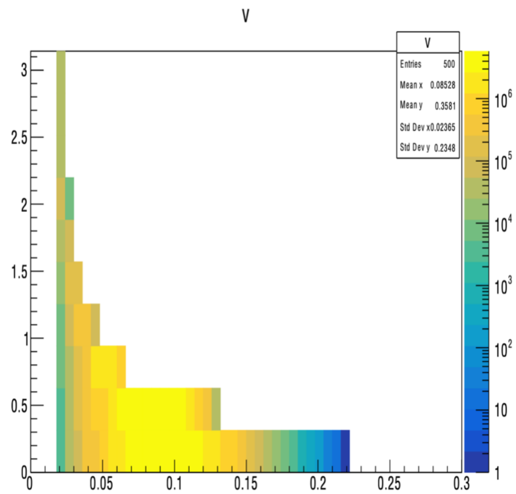
Unfold joint ${v_{2,a}+v_{2,b}}/{2}$ & $\Delta\psi$

$\Delta\psi$

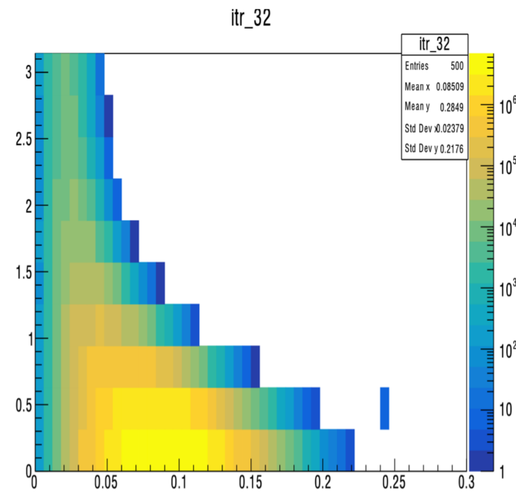
Smeared distribution



True distribution



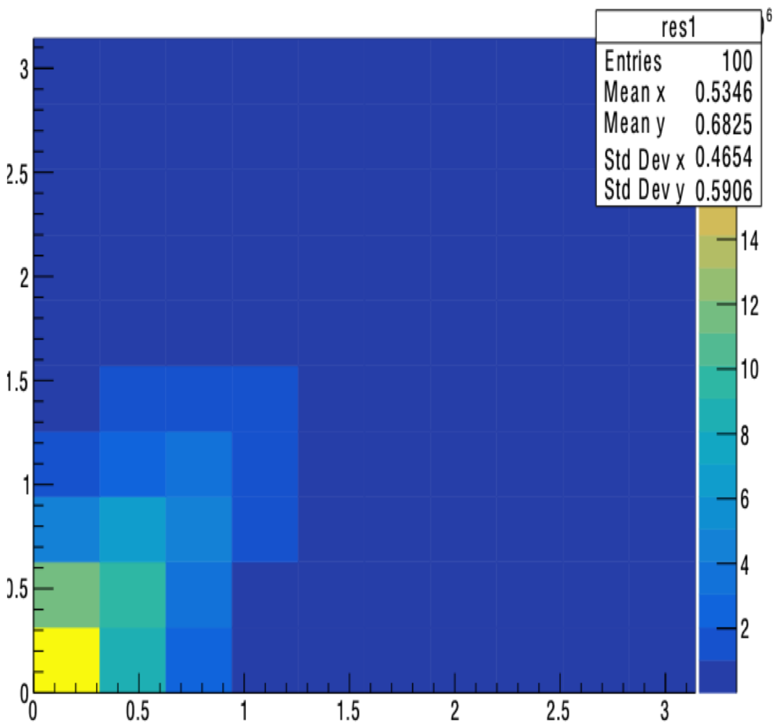
Unfold result
(32 iterations)



${v_{2,a} + v_{2,b}}/{2}$

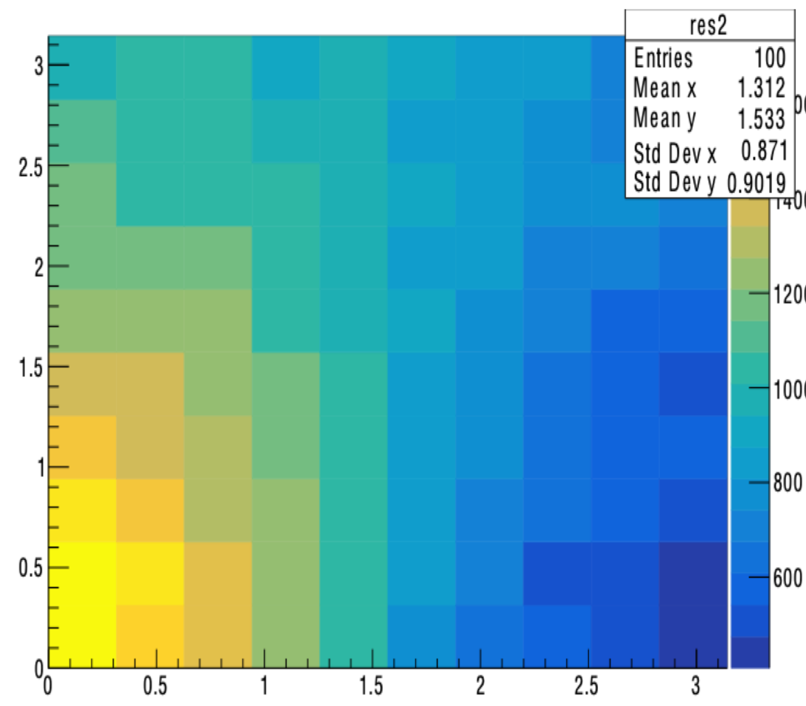
More dimensions give more information!

Response matrix of $\Delta\psi$ in 1D unfold



Response matrix of $\Delta\psi$ in 2D unfold (Joint with ${v_{2,a}+v_{2,b}}/{2}$)

And take a slice of small ${v_{2,a}+v_{2,b}}/{2}$.



More likely to be smeared.

3D unfolding

On running...